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A NOTE ON THE CENTRIFUGAL SCALING OF HORIZONTAL, ISOTHERMAL, LIQUID-GAS FLOWS WITHOUT MASS TRANSFER

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Abstract—The use of rotation to simulate increased gravity in scale models of horizontal liquid-gas flows is examined. The influences of Coriolis forces and natural gravity in the model are seen to be small provided that high rotation rates are used, and large length scale-down factors can then be achieved. The modelling of compressibility and gas-viscosity effects, however, is not normally possible and these must therefore be small in the original flow.

1. INTRODUCTION

In a recent article by this author (Chesters 1975) the criteria for dynamic similarity of isothermal, gas-liquid, two-phase flows without mass transfer were derived from the conventional equations governing such flows and the limitations of their validity discussed. The first three conditions for similarity of two flows, equality of Froude, Weber and liquid Reynolds numbers, were seen to require equality of the dimensionless liquid-property parameter $Q(=\rho_L\sigma^3/g\mu_L^4)$, together with characteristic length and velocity scales in the two flows proportional to $(\sigma/\rho_L g)^{1/2}$ and $(g\sigma/\rho_L)^{1/4}$ respectively (ρ_L, μ_L density and dynamic viscosity of the liquid, σ surface tension, g acceleration due to gravity). Taking g to be the same in each flow these requirements lead to

$$\left(\frac{\rho_L\sigma^3}{\mu_L^4}\right)_1 = \left(\frac{\rho_L\sigma^3}{\mu_L^4}\right)_2, \tag{1}$$

$$\frac{(x_c)_1}{(x_c)_2} = \frac{(\sigma/\rho_L)_1^{1/2}}{(\sigma/\rho_L)_2^{1/2}}, \tag{2}$$

$$\frac{(u_c)_1}{(u_c)_2} = \frac{(\sigma/\rho_L)_1^{1/4}}{(\sigma/\rho_L)_2^{1/4}}, \tag{3}$$

where x_c and u_c denote a characteristic length and velocity of the flow, respectively.

Although [1] is satisfied for many pairs of liquids (e.g. for water and trichloroethylene) the resulting values of $(x_c)_1/(x_c)_2$ do not in general differ greatly from unity (for water and trichloroethylene, $(x_c)_{tri}/(x_c)_w = 0.53$). A dramatic scale-down of a particular flow is therefore not possible in general. If, however, the value of g in the second flow is artificially increased by rotating the system, the situation is changed radically and large scale-down factors are obtainable.

The effects of increasing g in this way, including the complication of Coriolis forces, are examined below.

2. INFLUENCE OF g ON THE VARIOUS SCALE FACTORS

If the value of g is not assumed to be equal in the two flows the expressions for equality of Q and for the relative length and velocity scale factors become:

$$\left(\frac{\rho_L\sigma^3}{g\mu_L^4}\right)_1 = \left(\frac{\rho_L\sigma^3}{g\mu_L^4}\right)_2, \tag{4}$$

$$\frac{(x_c)_1}{(x_c)_2} = \frac{(\sigma/\rho_L g)_1^{1/2}}{(\sigma/\rho_L g)_2^{1/2}}, \tag{5}$$

$$\frac{(u_c)_1}{(u_c)_2} = \frac{(g\sigma/\rho_L)_1^{1/4}}{(g\sigma/\rho_L)_2^{1/4}}, \qquad [6]$$

These can be written more simply if the values of σ, μ_L, ρ_L, g, λ and u_c are expressed in units equal to their respective values in system 1. Since σ, μ_L, etc. then all acquire the value unity in flow 1, only symbols for these quantities in flow 2 remain and the suffix 2 can be omitted without confusion. Equations [4], [5] and [6] then become

$$\frac{\rho_L\sigma^3}{g\mu_L^4} = 1, \qquad [7]$$

$$x_c = (\sigma/\rho_L g)^{1/2}, \qquad [8]$$

$$u_c = (g\sigma/\rho_L)^{1/4} = (\sigma/\rho_L x_c)^{1/2} \qquad [9]$$

where the latter form of [9] is obtained by use of [8]. It is seen that a large value of g leads to a small value of x_c, that is, a large length scale-down factor.

The remaining criteria for similarity of the two flows are (Chesters 1975):

(a) equality of Euler number (either $p_G/\rho_G u_c^2$ or $p_{G,c}/\rho_L u_c^2$);
(b) equality of density ratio, $\rho_{G,c}/\rho_L$;
(c) equality of viscosity ratio, μ_G/μ_L;
(d) equality of system geometry, in which is included the ratio of the gas and liquid input flow rates.

(p absolute pressure, $p_{G,c}$ and $\rho_{G,c}$ gas pressure and density in some characteristic point in the flows.) Still using units based on the values of the quantities in system 1, (a) leads to

$$p_{G,c}/\rho_L u_c^2 = 1.$$

Making use of [9] and [8], this gives

$$p_{G,c} = (\rho_L g\sigma)^{1/2} = \sigma/x_c. \qquad [10]$$

(b) yields

$$\rho_{G,c}/\rho_L = 1.$$

With the help of the perfect gas law ($\rho_G = (\mathrm{MW})p_G/T$; MW molecular weight, T absolute temperature, with all quantities again in units equal to their values in flow 1), [10] and [8], this gives

$$\mathrm{MW} = T(\rho_L/g\sigma)^{1/2} = T\rho_L x_c/\sigma. \qquad [11]$$

Finally, (c) yields

$$\mu_G = \mu_L. \qquad [12]$$

From [9]–[12] it is seen that a small value of x_c (i.e. a large length scale-down factor) implies a large value of u_c and $p_{G,c}$ and a small value of MW and μ_G*. The former two (large velocity and

*Since the values of σ and ρ_L in [7] will not, in general, differ greatly from unity, a large value of g implies a small value of μ_L and hence, via [12], of μ_G.

pressure scale-up factors) can, in principle, always be provided but the latter two (small MW and μ_G for the model gas) are limited by the physical properties of available gases. If, therefore, very small values of x_c are required both condition [12] and either condition [11] or condition [10] (on which [11] is based) will have to be abandoned.

Equations [10] and [11] represent, respectively, the basic conditions (a) and (b). (a), equality of Euler number ($p_{G,c}/\rho_L u_c^2$), takes account of the compressibility of the gas and is important if either (i) appreciable expansion of the flow occurs over distances of interest, or (ii) pressure waves play an important role in the flow. The present method will be seen to be restricted in its applicability to approximately horizontal flows where both (i) and (ii) will indeed often be negligible (e.g. in their influence on the local pressure gradient or gas fraction in a pipeline containing liquid and high pressure gas). Neglect of (a) would therefore be an acceptable measure in many flows of interest.

(b), equality of $\rho_{G,c}/\rho_L$, takes account of the influences of gravity and of gas inertial forces on the liquid flow. These influences, which for example, largely determine the development of interfacial waves in large scale flows, are not likely to be negligible and should therefore be included in the scaling method.

If requirement [10], and hence [11], is relinquished, the gas pressure in the model can be freely chosen to allow requirement (b) to be satisfied. This requirement is now simply:

$$\mathrm{MW} = \rho_L T / p_{G,c}. \qquad [13]$$

Clearly it is sensible to choose MW (and thus $p_{G,c}$) as close as possible to the ideal values given by [11] and [10], since the greater the departure from these values the greater the influence of compressibility in the model, and at some point this influence will become appreciable. Ideally, one would test the model with two or more MW (and hence $p_{G,c}$) values to ascertain that the influence of compressibility is small, and to provide a correction for it if necessary (by extrapolation of the results to the ideal $P_{G,c}$ value).

The same argument as for compressibility can be applied to gas viscosity: its influence is probably slight in many large-scale flows and the failure to satisfy [12] is therefore probably not serious. However, it would be sensible to choose a model gas satisfying [12] as closely as possible and ideally more than one value of μ_G should be tested to ascertain its influence. Such tests could be combined with those on the influence of compressibility, which also involve a change of model gas.

3. USE OF ROTATION TO INCREASE THE VALUE OF g

The motion of a fluid with respect to steadily rotating axes can be described by the usual equations (continuity, state, Navier–Stokes and initial and boundary conditions) provided the body force per unit mass of the fluid, $\mathbf{F}$, is taken as

$$\mathbf{F} = \mathbf{g}_0 - 2\boldsymbol{\omega} \times \mathbf{u} - \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) \qquad [14]$$

where $\mathbf{r}$ is the position vector of the element of fluid concerned (the origin lying on the axis of rotation, figure 1) and $\boldsymbol{\omega}$ is the angular velocity vector of the system (Batchelor 1967). $\mathbf{g}_0$ is the natural gravitational acceleration, $-2\boldsymbol{\omega} \times \mathbf{u}$ is the Coriolis force per unit mass which is seen to be zero if the local velocity vector of the fluid, $\mathbf{u}$, is parallel to $\boldsymbol{\omega}$, and $-\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$ is the centrifugal force which is directed radially outwards with magnitude $R\omega^2$ (R = radius, i.e. perpendicular distance from the point concerned to the axis of rotation). Dividing [14] through by g_0 gives the value of g in [4]–[12] since g_0 represents the body force per unit mass in the original large-scale flow and F that in the (rotating) model flow:

$$\mathbf{g} = -\frac{\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})}{g_0} - \frac{2\boldsymbol{\omega} \times \mathbf{u}}{g_0} + \mathbf{k} \qquad [15]$$

where $\mathbf{k}$ is a unit vector in the direction of g_0.

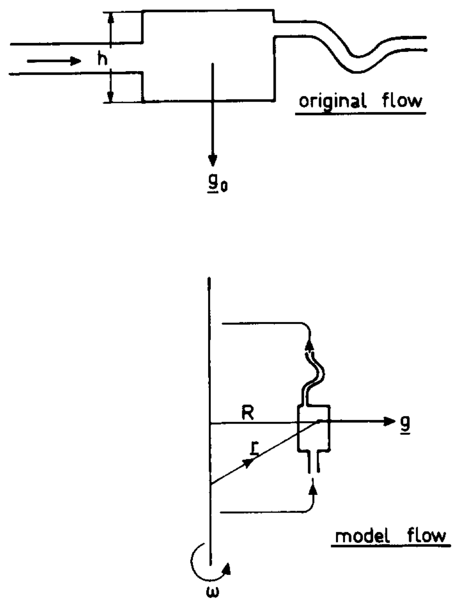

Figure 1. Original and scaled systems.

Except for very small values of ω the first (centrifugal) term in [15] will be seen to be much larger than the other two

$$\frac{|\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})|}{g_0}\gg 1, \qquad [16]$$

$$\frac{|\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})|}{|2\boldsymbol{\omega}\times\mathbf{u}|}\gg 1 \qquad [17]$$

and [15] becomes approximately

$$\mathbf{g}\simeq\frac{-\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})}{g_0}\simeq\frac{\omega^2\mathbf{R}}{g_0}. \qquad [18]$$

Thus for g to be approximately constant over the flow concerned, this flow may only have appreciable extension in the direction parallel to the axis of rotation and not in the radial direction. This confines the method to the scaling of approximately horizontal flows where, as in the model, the body force vector is perpendicular to the long dimension of the flow.

The extent to which g varies over the flow must now be considered. First, from [18], the fractional variation of g due to the radial extension of the flow is

$$\epsilon_1\simeq\frac{\Delta R}{R}\simeq\frac{h}{R}x_c \qquad [19]$$

where h is the vertical extension of the original flow (figure 1). If we suppose that R is chosen to be of the same order as h, [19] and [8] yield

$$\epsilon_1\sim x_c\sim g^{-1/2} \qquad [20]$$

since σ/ρ_L will in general be of the order of 1. The condition that this variation is negligible is then that

$$x_c\ll 1,\ \text{or} \qquad [21a]$$

$$g^{-1/2}\ll 1. \qquad [21b]$$

If R is chosen to be appreciably greater than h, [21] is unnecessarily strict.

The second departure from the constant g field given by [18] is that due to the Coriolis force, which produces a transverse component of g (either radial or circumferential or both, depending on the instantaneous local value of **u**). For a flow strictly parallel to the axis of rotation this force is absent but in general a lateral component of velocity, v, will exist, giving rise to a Coriolis component of relative magnitude

$$\epsilon_2 = \frac{2\omega v}{\omega^2 R} = \frac{2v}{\omega R}. \qquad [22]$$

Making use of [8], [9] and [18] and again taking $\sigma/\rho_L \sim 1$, [22] yields

$$\epsilon_2 \sim \frac{2u_c V}{(gg_0/R)^{1/2} R} \sim \frac{2V}{\sqrt{g_0 R}} \cdot g^{-1/4} \sim \frac{2V}{\sqrt{g_0 R}} x_c^{1/2} \qquad [23]$$

where V is the corresponding lateral component of velocity in the original flow. If we again suppose that $R \sim h$, the condition that Coriolis effects be negligible is therefore that

$$\frac{2V}{\sqrt{(g_0 h)}} \cdot g^{-(1/4)} \ll 1, \quad \text{or} \qquad [24a]$$

$$\frac{2V}{\sqrt{(g_0 h)}} \cdot x_c^{1/2} \ll 1. \qquad [24b]$$

Again, if R is chosen to be appreciably greater than h, [24] is unnecessarily strict.

Finally, a departure from the constant g field given by [18] occurs as a result of natural gravitation. The relative magnitude of this effect is

$$\epsilon_3 = \frac{g_0}{\omega^2 R} = g^{-1} \sim x_c^2 \qquad [25]$$

from [8]. ϵ_3 is thus of smaller magnitude than ϵ_1 for the case of $R \sim h$ and the condition that the effects of natural gravity be negligible is in general covered by [21].

In fact, if the axis of rotation is chosen to be vertical, the effect of natural gravity is to incline the g vector slightly to the axis of flow so that the situation really scaled is that of inclined flow (angle of inclination $= \epsilon_3 = g^{-1}$ radians). The influence of g_0 could therefore be removed by inclining the model flow to the axis of rotation by this same angle. This would, however, increase the influences of radial extension and Coriolis forces and might not be worth the gain.

It is noteworthy that by reversing the sense of rotation the Coriolis forces reverse direction. A system which is *not* symmetrical about a radial plane should then exhibit a change in overall characteristics (e.g. pressure drop) which would provide an indication of the magnitude of Coriolis effects. Likewise, if the flow system is inverted *and* the sense of rotation reversed, Coriolis forces are unchanged but the effect of natural gravity is reversed. An indication of the magnitude of natural gravity effects is then obtained.

4. AN EXAMPLE OF CENTRIFUGAL SCALING

As an example of the possible application of the above theory, consider a horizontal pipeline of diameter 30 cm containing a gas–liquid flow. It will be supposed that the liquid scalant is to be water and that the surface tension of the liquid in the pipeline is half that of water and density the same as that of water:

$$\sigma = 2, \quad \rho_L = 1. \qquad [26]$$

It will be supposed further that a length scale-down factor of 30 is required, and the feasibility of this will be examined. Equation [8] gives

$$1/30 = (2/g)^{1/2}; \quad g = 1800 \tag{27}$$

Taking $R = 30\,\text{cm}$ $(= h)$, [18] now gives

$$\omega = \sqrt{(1800\, g_0/30)} = 240\ \text{rad/sec.}$$

i.e. about 40 rev/sec: 2400 rev/min.

The required value of μ_L is given by [7]:

$$\mu_L = (8/1800)^{1/4} = 0.26. \tag{28}$$

Thus if the viscosity of the original liquid is 1/0.26 centipoise, water at 20°C (μ = 1 cp) is required. If the viscosity of the original liquid is 1 cp, water at about 100°C (μ = 0.26 cp) is required. At this temperature the vapour pressure is approx. 1 atm, but since the absolute pressure in the model will be very high the effect of mass transfer on the flow should still be slight (Chesters 1975).

According to [9]–[12] the required scale factors for velocity, pressure, gas molecular weight and gas dynamic viscosity are:

$$u_c = 7.7; \tag{29}$$

$$p_{G,c} = 60; \tag{30}$$

$$\text{MW} = 1/60; \tag{31}$$

$$\mu_G = 0.26. \tag{32}$$

For the sake of simplicity no account has been taken of the fact that T may not be exactly 1, just as variations in σ and ρ with temperature have been ignored. As anticipated in section 2, [32] cannot be satisfied whatever the pipeline gas concerned, and [31] only if the pipeline gas happens to have a molecular weight of at least 120. Otherwise a model gas with as low a molecular weight as possible would have to be used and the required pressure scale-up factor $p_{G,c}$ determined from [13].

The condition [21] is immediately seen to be fairly well satisfied. The condition [24] leads to

$$V \ll 4{,}7\ \text{m/sec.} \tag{33}$$

Since V is a typical *lateral* component of velocity in the pipeline it may be expected to be an order of magnitude smaller than a typical axial component, U. Equation [33] therefore implies

$$U \ll 47\ \text{m/sec} \tag{34}$$

which would certainly be satisfied in most such flows.

Finally, it is of interest to see how much less the mass flows of liquid and gas are in the model than in the pipeline. In units equal to the mass flow rate of either phase in the pipeline the mass flow rate of the same phase in the model is readily seen to be

$$m = \rho_L u_c x_c^2 = (\rho_L x_c^3 \sigma)^{1/2}, \tag{35}$$
$$= 1/116$$

in the present case.

5. FURTHER REMARKS

Although constructing a rotating model poses certain engineering difficulties there seems no reason why these could not be overcome: the rotation rate in the above example is within the normal range and most of the measurements carried out on static systems should be feasible in rotating systems.

A further interesting attribute of a rotating system is that by varying the rotation rate the influence of the Froude number can be ascertained since this is the only dimensionless group which is affected.

Finally it is noted that the foregoing theory is based on the conventional equations for isothermal, liquid–gas, two-phase flows without mass transfer and since such equations do not adequately describe the last stages of rupture of thin films or filaments, the applicability of the present similarity theory to flows in which such processes occur cannot yet be taken for granted (Chesters 1975).

6. CONCLUSIONS

1. By artificially increasing gravity by use of a rotating system, large length scale-down factors can be achieved in the modelling of horizontal, isothermal, liquid–gas flows without mass transfer.

2. In order for the analogy between centrifugal force in the model and natural gravity in the original system to be complete, it is necessary that the influences of Coriolis forces and of natural gravity be negligible in the model. Also the radial extension of the model flow must be small in comparison with its mean radius (measured from the axis of rotation). These requirements are met if the scale-down factor (and hence the rotation rate) is sufficiently large: typically $\gtrsim 10$ times.

3. The use of such large scale-down factors, however, in general prohibits the modelling of compressibility and gas-viscosity effects and these must therefore be slight in the original flow if the method is to be applicable.

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